

TASK

Implement reduced rank regression as a simple R function.

SOLUTION

The R code is displayed below.

```
ReducedRankRegression <- function(Xraw,Y,Rank) {

  # This code implements a two-stage reduced-rank estimation.
  # First, we estimate the covariance matrix Sigma of the residuals
  # via OLS. Second, we engage reduced-rank regression supplying Sigma
  # as an input.

  Xraw      <- as.matrix(Xraw);
  Y         <- as.matrix(Y);

  # STAGE 1
  N         <- dim(Y)[1];
  q         <- dim(Y)[2];
  X         <- cbind(rep(1,N), Xraw);
  OlsRank   <- dim(X)[2];
  EffectiveRank <- Rank + 1;
  Bols      <- t( solve( t(X)%*%X ) %*% t(X)%*%Y );

  Sigma     <- matrix(0,q,q);
  Sxx       <- matrix(0,q+1,q+1);
  Sxy       <- matrix(0,q+1,q);
  Syy       <- matrix(0,q,q);
  for(t in 1:N) {
    Sxx     <- Sxx + X[t,]%*%t(X[t,]);
    Sxy     <- Sxy + X[t,]%*%t(Y[t,]);
    Syy     <- Syy + Y[t,]%*%t(Y[t,]);

    Resid   <- Y[t,] - Bols%*%X[t,];
    Sigma   <- Sigma + Resid%*%t(Resid);
  }
  Sxx      <- Sxx / N;
  Sxy      <- Sxy / N;
  Syx      <- t(Sxy);
  Syy      <- Syy / N;
  Sigma    <- Sigma / (N-OlsRank);

  # STAGE 2
  Gamma    <- solve(Sigma);
  HalfGamma <- chol(Gamma);
  H        <- HalfGamma%*%Syx%*%solve(Sxx)%*%Sxy%*%HalfGamma;
  SVDoutput <- svd(H);
  RelevEigenVectors <- SVDoutput$u[,1:EffectiveRank];

  MiddleSum <- matrix(0,q,q);
  for(j in 1:EffectiveRank) {
    MiddleSum <- MiddleSum +
  RelevEigenVectors[,j]%*%t(RelevEigenVectors[,j]);
  }
}
```

```
B          <- solve(HalfGamma)*%*%MiddleSum*%*%HalfGamma %*%  
Syx*%*%solve(Sxx);  
  
list(B=B, Sigma=Sigma, Bols=Bols);  
  
}
```

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